USN | | | | | | | | | | 15MAT11

First Semester B.E. Degree Examination, July/August 2022
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(06 Marks)**

b. Find the angle of intersection of the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$. **(05 Marks)**

c. Show that the radius of curvature at any point of : $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$ is $4a\cos(\theta/2)$. **(05 Marks)**

OR

2 a. Find the n^{th} derivative of $\dfrac{x+1}{x-1} + e^{-2x}\cos^2 x$. **(06 Marks)**

b. Find the pedal equation of $r^n = a^n \cos n\theta$. **(05 Marks)**

c. Show that radius of curvature on the curve $y^2 = \dfrac{a^2(a-x)}{x}$ at (a, 0) is $\dfrac{a}{2}$. **(05 Marks)**

Module-2

3 a. Expand $\log_e x$ in powers of $(x-1)$ upto fifth degree term. **(06 Marks)**

b. If $u = \tan^{-1}\left[\dfrac{x^3+y^3}{x-y}\right]$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$ **(05 Marks)**

c. If $u = x^2+y^2+z^2$, $v = xy+yz+zx$ and $w = x+y+z$ find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **(05 Marks)**

OR

4 a. Evaluate $\lim\limits_{x\to\frac{\pi}{4}} (\tan x)^{\tan 2x}$. **(06 Marks)**

b. If $Z = f(x, y)$ where $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that $\dfrac{\partial z}{\partial u} - \dfrac{\partial z}{\partial v} = x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y}$. **(05 Marks)**

c. If $V = (x^2+y^2+z^2)^{-1/2}$, prove that $V_{xx} + V_{yy} + V_{zz} = 0$. **(05 Marks)**

Module-3

5 a. A particle moves along the curve $x = t^3+1$, $y = t^2$, $z = 2t+3$ where t is the time. Find the Components of its velocity and acceleration at t = 1 in the direction $\hat{i}+\hat{j}+3\hat{k}$. **(06 Marks)**

b. If $\vec{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$, find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$ at (1, 2, 3). **(05 Marks)**

c. For any scalar point function ϕ and a vector point function $\vec{A}$, show that $\text{div}(\phi\vec{A}) = \phi(\text{div}\,\vec{A}) + \text{grad}\,\phi\cdot\vec{A}$. **(05 Marks)**

OR

6 a. If $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{R}| = r$, show that $\nabla r^n = nr^{n-2}\vec{R}$. (06 Marks)

b. Show that for any scalar point function ϕ, $\nabla_n X(\nabla\varphi) = 0$. (05 Marks)

c. If $u = x^2 + y^2 + z^2$, $\vec{V} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\operatorname{div}(u\vec{V}) = 5u$. (05 Marks)

Module-4

7 a. Obtain the reduction formula for $\int \sin^n dx$ (06 Marks)

b. Solve the differential equation : $(x^4 - 2xy^2 + y^4)\,dx - (2x^2y - 4xy^3 + \sin y)\,dy$. (05 Marks)

c. Find the orthogonal trajectories of the family of coaxial circles $x^2 + y^2 + 2\lambda x + c = 0$, λ being the parameter. (05 Marks)

OR

8 a. Evaluate $\int_0^{2a} \frac{x^3}{\sqrt{2ax - x^2}}\,dx$ (06 Marks)

b. Solve $x\frac{dy}{dx} + y = x^3y^6$. (05 Marks)

c. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original? (05 Marks)

Module-5

9 a. Find the rank of the matrix :

$$\begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}.$$ (06 Marks)

b. Using Gauss Seidel method solve :

$20x + y - 2z = 17$

$2x - 3y + 20z = 25$

$3x + 20y - z + 18 = 0$

in three iterations with $(x_0, y_0, z_0) = (0, 0, 0)$. (05 Marks)

c. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. (05 Marks)

OR

10 a. Solve the system of equations :

$x + y + z = 9$

$2x - y + 2z = 15$

$3x + 2y + z = 12$

by Gauss elimination method. (06 Marks)

b. Using Rayleigh's power method find the dominant eigen value of

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

in five iterations, choosing $X_0 = [1\ \ 0\ \ 0]^T$. (05 Marks)

c. Show that the transformation

$y_1 = 2x_1 + x_2 + x_3$

$y_2 = x_1 + x_2 + 2x_3$

$y_3 = x_1 - 2x_3$

is regular. Find the inverse transformation. (05 Marks)